Mixed Synchronization of Chaotic Systems with Uncertain Parameters

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Abstract: In this paper, mixed synchronization of chaotic system with uncertain parameters is investigated. Mixed synchronization is more general that includes function projective synchronization, complete synchronization, anti-synchronization and projective synchronization as its special cases. Based on the stability of linear system and the adaptive control technique, a general mathematical method for designing the controllers is proposed in order to achieve the mixed synchronization between chaotic systems with fully uncertain parameters.

1. Introduction

During the last two decades, synchronization have received a significant attention among scientists from various different fields [1-3]. Many kinds of synchronization have been proposed in dynamical systems. Projective synchronization is the most noticeable one because of its proportional feature between the synchronized dynamical states and hence it has received extensive research. More recently, Shen et al. [4,5] considered the function projective synchronization problem of chaotic systems, in which the drive and the response systems can synchronize up to a desired scaling function. By summarizing the previous results, it is interesting to ask if that the response system has scaling factor matrix and the drive system has scaling function matrix during the synchronization. On the other hand, the synchronization and identification of chaotic systems with uncertainties is a more essential work for research[6,7].

2. The Scheme for mixed synchronization

Consider the drive system and the response system described as follows

$$\dot{\mathbf{x}}(t) = F(\mathbf{x}), \tag{1}$$

$$\dot{\mathbf{y}}(t) = G(\mathbf{y}) + U(t, \mathbf{x}, \mathbf{y}), \tag{2}$$

where $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T, \mathbf{y} = (y_1, y_2, \cdots, y_n)^T \in \mathbf{R}^n$ are state variables of the drive system (1) and the response system (2), respectively. $F(\mathbf{x})$ and $G(\mathbf{y})$ are the continuous vector functions, $U(t, \mathbf{x}, \mathbf{y})$ is a controller function that needs to be designed.

Let $\mathbf{e} = \boldsymbol{\alpha}\mathbf{y} - \mathbf{k}(t)\mathbf{x}$ is the synchronization error vector. Hence the error dynamics system takes the form $\dot{\mathbf{e}} = \boldsymbol{\alpha}\dot{\mathbf{y}} - \dot{\mathbf{k}}(t)\mathbf{x} - \mathbf{k}(t)\dot{\mathbf{x}}$. One can get

$$\begin{aligned} \dot{\mathbf{e}} &= \boldsymbol{\alpha}\dot{\mathbf{y}} - \dot{\mathbf{k}}(t)\mathbf{x} - \mathbf{k}(t)\dot{\mathbf{x}} \\ &= \boldsymbol{\alpha}[f_2(\mathbf{y}) + F_2(\mathbf{y})\boldsymbol{\Omega} + U(t, \mathbf{x}, \mathbf{y})] - \dot{\mathbf{k}}(t)\mathbf{x} - \mathbf{k}(t)(f_1(\mathbf{x}) + F_1(\mathbf{x})\boldsymbol{\Phi}) \end{aligned} \tag{3}$$

Let $\hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{\Omega}}$ denote the estimations of the uncertain parameters $\boldsymbol{\Phi}, \boldsymbol{\Omega}$. We design the controller as following:

$$U(t, \mathbf{x}, \mathbf{y}) = \boldsymbol{\alpha}^{-1}[\mathbf{k}(t)(F_1(\mathbf{x})\hat{\boldsymbol{\Phi}} + f_1(\mathbf{x})) + \dot{\mathbf{k}}(t)\mathbf{x} - \mathbf{P}\mathbf{e}] - F_2(\mathbf{y})\hat{\boldsymbol{\Omega}} - f_2(\mathbf{y}). \tag{4}$$

The parameter update laws are determined as

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$$\begin{cases} \dot{\hat{\mathbf{\Phi}}} = -[\mathbf{k}(t)F_1(\mathbf{x})]^T \mathbf{e} \\ \dot{\hat{\mathbf{\Omega}}} = [\mathbf{\alpha} F_2(\mathbf{y})]^T \mathbf{e} \end{cases} \tag{5}$$

3. Examples of applications

The drive system and the fractional-order Chen chaotic system, as the response system, which is described as:

$$\begin{cases} \dot{x}_1(t) = a(x_2 - x_1) \\ \dot{x}_2(t) = x_1x_3 - x_2 \\ \dot{x}_3(t) = b - x_1x_2 - cx_3, \end{cases} \tag{6}$$

$$\begin{cases} D_*^q y_1(t) = d(y_2 - y_1) + u_1 \\ D_*^q y_2(t) = (f-d)y_1 - y_1y_3 + fy_2 + u_2 \\ D_*^q y_3(t) = y_1y_2 - hy_3 + u_3, \end{cases} \tag{7}$$

When $d = 35, h = 3, f = 28$ and $q = 0.9$, system (9) exhibits chaotic behavior as shown in Fig.1.

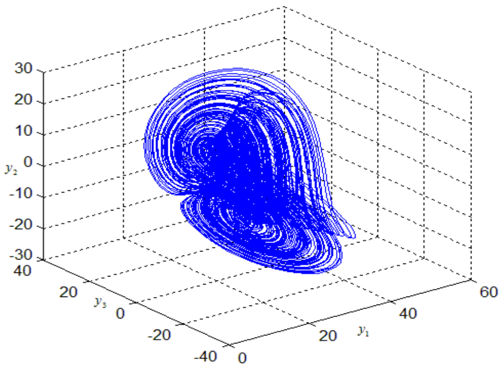

Fig.1. Three-dimensional plot of the trajectory of the fractional-order Chen system

We first choose the scaling factor and the scaling function matrices to be diagonal matrices as the following:

$$\mathbf{\beta} = \begin{bmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{bmatrix}, \qquad \mathbf{M}(t) = \begin{bmatrix} \phi_1(t) & 0 & 0 \\ 0 & \phi_2(t) & 0 \\ 0 & 0 & \phi_3(t) \end{bmatrix}$$

Theorem 1 For the given constant scaling factors $\beta_i(i = 1,2,3)$ and the scaling functions $\phi_i(t)(i = 1,2,3)$, the adaptive mixed synchronization will be achieved by the following controller and the adaptive law of the unknown parameters as

$$\begin{cases} u_1 = \dfrac{1}{\beta_1}[D_*^q(\phi_1(t)x_1) - e_1] - \hat{d}(y_2 - y_1) \\ u_2 = \dfrac{1}{\beta_2}[D_*^q(\phi_2(t)x_2) - e_2)] - (\hat{f} - \hat{d})y_1 + y_1y_3 - \hat{f}y_2 \\ u_3 = \dfrac{1}{\beta_3}[D_*^q(\phi_3(t)x_3) - e_3)] - y_1y_2 + \hat{h}y_3. \end{cases} \tag{8}$$

$$\begin{cases} D_*^q e_d = \beta_1(y_2 - y_1)e_1 - \beta_2 y_1 e_2 \\ D_*^q e_f = \beta_2(y_1 + y_2)e_2 \\ D_*^q e_h = -\beta_3 y_3 e_3. \end{cases} \tag{9}$$

Where $e_d = d - \hat{d}, e_h = h - \hat{h}, e_f = f - \hat{f}$ are the corresponding parameter errors.

The predictor-corrector algorithm is utilized to solve the differential equations with fractional-order. The simulation results are described as follows:

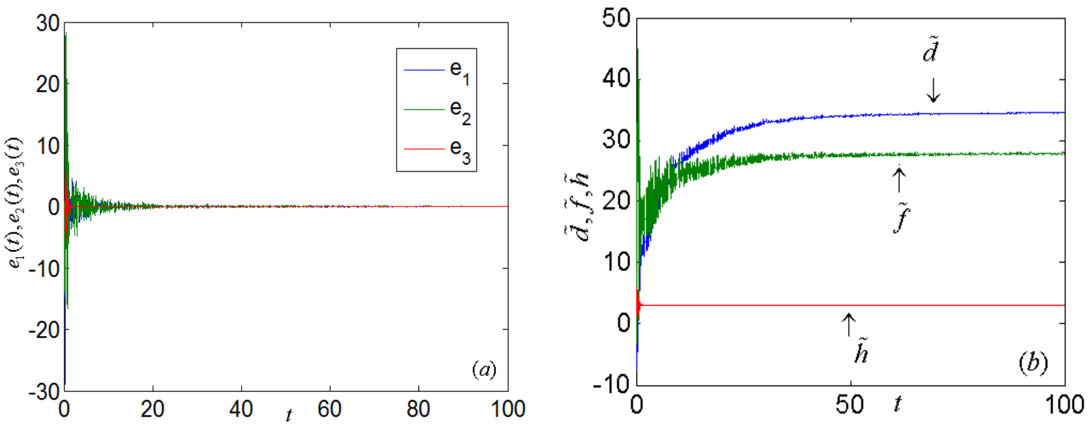

Fig.2 (a) Time evolution of errors (b) Time evolution of parameters estimation for system (9).

4. Conclusion

In this paper, the mixed function projective synchronization of chaotic systems with uncertain parameters is discussed. By means of the stability theory and the adaptive control technique, synchronization schemes are derived via an appropriate controller and the adaptive laws to realize the mixed function projective synchronization for chaotic systems. Numerical simulations show that the effectiveness and feasibility of the controllers and the parameter identification laws.

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